Double Auctions with No-Loss Constrained Traders*

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Abstract

Do hard budget constraints work in favour or against truthtelling in double auctions? McAfee (1992) constructed a simple double auction mechanism (MDA), which is strategyproof and minimally inefficient, but may resort to dual prices, where the difference between prices is channelled as a surplus to the market maker, preventing MDA from achieving a balanced budget. We construct a variant of MDA in which No-Loss Constraints play a major positive role. Our variant of MDA is also strategyproof, as efficient as MDA but improves on it by achieving a balanced budget via always having a uniform price.

Keywords: No-Loss Constraint (NLC), double auctions (DA), uniform price, efficiency, balanced budget, sunspots.
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1 Introduction

Trade in most important homogeneous-goods markets has been governed primarily by periodic systems in the form of a 'double auction' (DA) for more than hundred-fifty years. They are used to open many continuous markets such as the New York Stock Exchange (NYSE), the Tokyo Stock Exchange and the Chicago Mercantile Exchange. DAs collect bids and asks/offers from traders, implicitly construct supply and demand curves, announce a market-clearing price, and execute the indicated trades. As such, DA comes closest to operationalising Marshall's supply-demand diagram among all institutions that mediate trade.

These markets are always in flux where investors need to make quick decisions, many of which push them to eventual losses. In finance, in particular, it is well recognised that investors are prone to becoming extra cautious in trying to avoid losses at almost all costs. This can encourage traders

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to play it safe by never bidding above their private valuations and never asking below their private costs. Hard budget constraints may be another reason for such cautious behaviour as traders may have no means to face any amount of ex-post monetary loss. In this paper we use the weakest form of such a constraint. In particular, while we allow traders to bid above their valuations or ask below their costs, they are constrained from incurring any loss ex-post. Such traders will be called No-Loss-Constrained (NLC) traders and will be the focus of this paper.

The main issue in the literature on DA is the tension between efficiency, incentive compatibility and balanced-budget. As the famous impossibility result of Myerson and Satterthwaite (1983) suggests, under fairly general conditions, any double auction that is incentive compatible for both the buyers and the sellers must yield a deficit if it wants to achieve ex-post efficiency. It is also well known, for example, that the Vickrey-Clarke-Groves mechanism (Vickrey (1965), Clarke (1971) and Groves (1973)) achieves efficiency, individual rationality and incentive-compatibility but runs a deficit. A question that becomes central then is: what can we achieve with a balanced budget DA? Chatterjee and Samuelson (1983) proposed a mechanism for bilateral trading where the buyer pays the seller an average of their bids and nothing otherwise. The mechanism is balanced budget and strategyproof but not efficient. McAfee (1992), for instance, devised an ingenious yet simple DA mechanism (MDA hereafter for the McAfee Double Auction) with single-unit demand and supply that achieves strategyproofness, and is minimally inefficient (in the sense that it leaves out at most the least efficient trading pair). Indeed it also avoids a budget deficit, but *fails* to have a balanced budget via a *uniform price*, since MDA may use dual prices, where the difference between prices is channelled as a surplus to the market maker.[1]

In this delicate context, it is far from clear whether NLC constraints alleviate this problem or not. This is because it is not obvious how incentives to misreport may get affected in the presence of NLC. We find that in a world with NLC traders, there exists a simple variant of MDA that *improves* the performance of the mechanism: it not only remains strategyproof and equally efficient as MDA but also has a uniform price - via a price lottery - and hence a balanced budget. In a sense, this result highlights the crucial link of NLC to strategyproofness when DAs are concerned. Indeed, MDA fails to be strategyproof if one uses our price lottery to achieve a uniform price in a non-NLC world. As a consequence, since the surplus that would go to the market maker in MDA gets channelled either to the buyers or to the sellers in our scheme (thus benefitting both sides in expected sense without hurting either in ex-post sense), NLC traders would strictly prefer our scheme to MDA.[2] It is important to note that, instead of assuming NLC, if we assume stricter

[1]Matters get more complex with multiunit demands and supplies (as in Loertscher and Mezzetti (2014) or with interdependent values (as in Kojima and Yamashita (2014)). For an excellent survey on the design of two-sided markets, see Loertscher, Marx and Wilkening (2015).

[2] We are thankful to Preston McAfee for sharing this observation with us. As in a world with NLC traders, MDA maintains all its properties, it follows that in such a world our mechanism is more efficient than MDA in ex-ante sense if welfare is utilitarian. However, as explained in Section 4, our mechanism ceases to be strategyproof in a world where traders are not NLC.

restrictions where buyers never bid above their valuations or sellers never ask below their costs, then our results continue to hold.[3] Lastly, as we discuss further in the paper, randomness of the price does not necessarily have to be a mechanism-design instrument. Instead, one may think of a situation where the market maker is endowed with a private type that depicts whether she is "pro-buyer" or "pro-seller". The randomised price can then be determined by the realisation of this type, giving rise to sunspots.

The rest of the paper is structured as follows. In Section 2 we describe the DA market and in Section 3 we define our DA mechanism. In Section 4 we state and prove our main result, followed by a discussion. The paper provides its concluding remarks in Section 5.

2 The framework

The market: Following McAfee (1992), consider a market with N buyers, indexed by $i \in \mathcal{N}^B = \{1, ..., N\}$, and M sellers, indexed by $j \in \mathcal{N}^S = \{1, ..., M\}$. Each buyer i has a *privately observed* value b_i for a single unit of a *homogenous* good while each seller j has a *privately observed* cost s_j for providing a single unit of the good. We assume unrestricted domains for values and costs as long as they are non-negative, that is, $b_i, s_j \geq 0$ for all i, j. We denote by (b, s) a profile of these private values and costs (*profile*, in short). A buyer with value b_i who obtains the object at price p earns a payoff of $u_i = b_i - p$ while a seller with cost s_j who sells the object earns a payoff of $v_j = p - s_j$. Both buyers and sellers earn zero when they are not able to trade. Buyers and sellers privately report their bids and asks to the market maker who employs a Double Auction (DA) to decide which buyers and sellers trade and at what prices. We denote a DA mechanism by $\mathcal{M}$ and the reported profile by the pair (β, σ) where $\beta = (\beta_1, ..., \beta_N)$ and $\sigma = (\sigma_1, ..., \sigma_M)$ are the reports of the buyers and sellers respectively.

Volume of trade: Given a profile (b, s) and the order statistics $b_{(1)} > ... > b_{(N)}$ and $s_{(1)} < ...., s_{(M)}$, the *efficient* number of trades is the number $k \leq \min\{N, M\}$ such that $b_{(k)} \geq s_{(k)}$ and $b_{(k+1)} < s_{(k+1)}$. As in McAfee we too will call the traders identified with the number k as our *marginal traders* and maintain that a DA mechanism dictates that a trade still occurs even if it produces exactly zero surplus to either party. A *minimally inefficient* mechanism is one where the number of trades is $k - 1$ with some probability and k with the remaining probability.

Price rule: Denote by $\mathcal{T}_{(\beta,\sigma)} = \mathcal{B}_{(\beta,\sigma)} \cup \mathcal{S}_{(\beta,\sigma)}$ the set of buyers $(\mathcal{B}_{(\beta,\sigma)})$ and sellers $(\mathcal{S}_{(\beta,\sigma)})$ who trade at the reported profile (β, σ). Let $\pi_{\beta,\sigma} : \mathcal{T}_{(\beta,\sigma)} \to \Delta(\mathbb{R}_+)$ be the *price rule* that allocates a price lottery $\pi_{\beta,\sigma}(t)$ (that is a probability distribution over $\mathbb{R}$) to trader $t \in \mathcal{T}_{(\beta,\sigma)}$ at profile (β, σ). For $t \in \mathcal{T}_{(\beta,\sigma)}$, let $p_{\beta,\sigma}(t)$ be the realized price for t under the lottery $\pi_{\beta,\sigma}(t)$. Then t pays a price

[3] As pointed out by a referee, there are many implementations in experiments of private-values markets in which bidding above value or below cost was ruled out by the decision interface.

$p_{\beta,\sigma}(t)$ and obtains a monetary payoff $b_t - p_{\beta,\sigma}(t)$ if $t \in \mathcal{B}_{(\beta,\sigma)}$ while if $t \in \mathcal{S}_{(\beta,\sigma)}$, then t receives this price $p_{\beta,\sigma}(t)$ and obtains a monetary payoff of $p_{\beta,\sigma}(t) - s_t$.

Properties of the mechanism: We say that $\mathcal{M}$ has a *uniform price rule at profile* (β,σ) if $\pi_{\beta,\sigma}(t) = \pi_{\beta,\sigma}(t')$ for each $t, t' \in \mathcal{T}_{(\beta,\sigma)}$ and $\pi_{\beta,\sigma}(t)$ determines the 'market' price from a draw, common to all buyers and sellers. We call $\mathcal{M}$ a *uniform price* DA if $\mathcal{M}$ has a uniform price rule at each reported profile (β,σ). We say that $\mathcal{M}$ has a *balanced budget* if the total monetary amount paid by all buyers who trade equals the total monetary amount received by all sellers who sell at any reported reported profile (β,σ), that is, $\forall(\beta,\sigma)$, $\sum_{t\in\mathcal{B}_{(\beta,\sigma)}} p_{\beta,\sigma}(t) = \sum_{t\in\mathcal{S}_{(\beta,\sigma)}} p_{\beta,\sigma}(t)$, for each $p_{\beta,\sigma}(t)$ in the support of $\pi_{\beta,\sigma}(t)$, $t \in \mathcal{T}_{(\beta,\sigma)}$. We say that $\mathcal{M}$ is *No-Loss Constrained* (NLC), if, ex post, no trader is worse off by participating in the double auction than by not participating. As no participation yields zero monetary payoff to the trader, then NLC simply means that in each state of the world, each trader must obtain a non-negative monetary payoff: for each buyer $t \in \mathcal{B}_{(\beta,\sigma)}$, we have $p_{\beta,\sigma}(t) \le b_t$ for each $p_{\beta,\sigma}(t)$ in the support of $\pi_{\beta,\sigma}(t)$ and for each seller $t' \in \mathcal{S}_{(\beta,\sigma)}$, we have $p_{\beta,\sigma}(t') \ge s_t$ for each $p_{\beta,\sigma}(t')$ in the support of $\pi_{\beta,\sigma}(t')$. We now define strategyproofness under NLC. For a given report (β,σ), the expected payoff of buyer $t \in \mathcal{B}_{(\beta,\sigma)}$ with true valuation b_t is $U_t(\beta,\sigma) = \int_{p_{\beta,\sigma}(t)\in\mathbb{R}_+} (b_t - p_{\beta,\sigma}(t))\pi_{\beta,\sigma}(t)\mathrm{d}p_{\beta,\sigma}(t)$, the expected payoff of seller $t \in \mathcal{S}_{(\beta,\sigma)}$ with true cost s_t is $V_t(\beta,\sigma) = \int_{p_{\beta,\sigma}(t)\in\mathbb{R}_+} (p_{\beta,\sigma}(t) - s_t)\pi_{\beta,\sigma}(t)\mathrm{d}p_{\beta,\sigma}(t)$, while $U_t(\beta,\sigma) = V_t(\beta,\sigma) = 0$ for all $t \notin \mathcal{T}_{(\beta,\sigma)}$. The mechanism is NLC-buyer-manipulable at a reported profile (β,σ) if the mechanism respects the NLC conditions described above and there exists a buyer $t \in \mathcal{N}^B$ and a bid $\beta_t \neq b_t$ such that $U_t((\beta_{-t},\beta_t),\sigma) > U_t((\beta_{-t},b_t),\sigma_t)$. Similarly, the mechanism is NLC-seller-manipulable at a reported profile (β,σ) if the mechanism respects the NLC conditions described above and there exists a seller $t \in \mathcal{S}^B$ and an ask $\sigma_t \neq s_t$ such that $V_t(\beta,(\sigma_{-t},\sigma_t)) > V_t(\beta,(\sigma_{-t},s_t))$. The mechanism is NLC-strategyproof if there does not exist a reported profile (β,σ) that is NLC-manipulable by any buyer or seller.

Our objective is to find a minimally inefficient NLC-strategyproof mechanism that is budget-balanced.

3 The mechanism

Consider the following mechanism denoted by $\mathcal{M}_q$: for any arbitrary report profile (β,σ), consider the order statistics $\beta_{(1)} > ... > \beta_{(N)}$ and $\sigma_{(1)} < ... < \sigma_{(M)}$ where the subscripts (i) and (j) represent the ranks for the buyers and sellers in these orderings respectively. Fix any probability q, with $0 < q < 1$, and denote by p the uniform price. Of course, uniform price immediately makes the mechanism budget-balanced.

Criterion 1 : If $k = \min\{N, M\}$, then $\pi_{\beta,\sigma}$ yields $p = \beta_{(k)}$ with probability q and $p = \sigma_{(k)}$ with probability $1-q$ and at the realized price all buyers with bids $\ge \beta_{(k-1)}$ and all sellers with asks $\le \sigma_{(k-1)}$ trade while the remaining traders stay out; volume of trade $= k-1$;

Criterion 2 : If $k < \min\{N, M\}$ and

(A) $\beta_{(k)} \geq \frac{\beta_{(k+1)}+\sigma_{(k+1)}}{2} \geq \sigma_{(k)}$, then $\pi_{\beta,\sigma}$ yields $p = \frac{\beta_{(k+1)}+\sigma_{(k+1)}}{2}$ with probability 1 at which all buyers with bids $\geq \beta_{(k)}$ and all sellers with asks $\leq \sigma_{(k)}$ trade while all other agents stay out; volume of trade $= k$;

(B) either $\beta_{(k)} < \frac{\beta_{(k+1)}+\sigma_{(k+1)}}{2}$ or $\frac{\beta_{(k+1)}+\sigma_{(k+1)}}{2} < \sigma_{(k)}$, then $\pi_{\beta,\sigma}$ yields $p = \beta_{(k)}$ with probability q and $p = \sigma_{(k)}$ with probability $1 - q$ and at the realized price all buyers with bids $\geq \beta_{(k-1)}$ and all sellers with asks $\leq \sigma_{(k-1)}$ trade while all other players stay out; If $k = 1$ then no trade takes place; volume of trade $= k - 1$;

REMARK **1 (The McAfee Double Auction (MDA))** *For a given reported profile (β, σ), define $p_0 = \frac{\beta_{(k+1)}+\sigma_{(k+1)}}{2}$. The MDA and $\mathcal{M}_q$ coincide at each profile of reports (β, σ) such that $\beta_{(k)} \geq p_0 \geq \sigma_{(k)}$ where in both mechanisms all top k value buyers and bottom k cost sellers trade. On the other hand, if $p_0 \notin [\sigma_{(k)}, \beta_{(k)}]$ the quantity traded is $k - 1$ in both the mechanisms, but in the MDA, all buyers pay $\beta_{(k)}$ while all sellers are paid $\sigma_{(k)}$. In case of $\mathcal{M}_q$, the realized price is uniform through the price lottery $\pi_{(\beta,\sigma)}$ over the binary set $\{\sigma_{(k)}, \beta_{(k)}\}$. Our benchmark for minimal inefficiency is MDA.*

REMARK **2 (Price-uncertainty and Sunspots)** *A useful modification of the mechanism $\mathcal{M}_q$ would be to incorporate a binary state of the world which is observed by the market maker but not by the participants. At its simplest, it could be the type of the market maker, who could be "pro-buyer" (who sets $p = \sigma_{(k)}$) or "pro-seller" (who sets $p = \beta_{(k)}$) where now, q is the ex-ante probability that the market-maker is "pro-seller." This would eliminate uncertainty as an instrument and would mean that the mechanism is in fact not random at all. The price would indeed be uncertain from the perspective of the traders but this uncertainty would be more as a response to 'sunspot' variables (viz. the market-maker's type), that is, to random events that have nothing to do with economic fundamentals.*[4]

4 The result

THEOREM **1** *For each $q \in (0, 1)$, the budget-balanced uniform-price $\mathcal{M}_q$ is NLC-strategyproof and as efficient as MDA.*

Proof: Volumes of trade under each criterion in the definition of $\mathcal{M}_q$ prove that the efficiency of $\mathcal{M}_q$ is that of MDA. So we prove NLC-strategyproofness. Fix an arbitrary $q \in (0, 1)$. Denote by $(\beta, \sigma)_i = ((\beta_{-i}, b_i), \sigma)$ a profile where buyer i bids truthfully while the bids and the asks of others are arbitrary. Similarly, denote by $(\beta, \sigma)_j = ((\beta, (\sigma_{-j}, s_j))$ a profile where seller j bids truthfully while the bids and the asks of others are arbitrary.

[4]We thank an anonymous referee for suggesting this to us.

Case 1 $k = \min\{N, M\}$ (Criterion 1): Consider profile $(\beta, \sigma)_i$ and let $k = \min\{N, M\} = N$ (the argument is analogous when $\min\{N, M\} = M$). Then $\pi_{(\beta,\sigma)_i}$ is a lottery with $p = \beta_{(N)}$ with probability q and $p = \sigma_{(N)}$ with probability $1 - q$ but the buyer with bid $\beta_{(N)}$ and the seller with ask $\sigma_{(N)}$ stay out while all other agents trade. It is not possible that buyer i with $b_i > \beta_{(N)}$ can affect the price without i becoming a marginal trader and earning 0, which is less than i's payoff of $b_i - \beta_{(N)} > 0$ that occurs with probability q or $b_i - \sigma_{(N)} > 0$ that occurs with probability $1 - q$. So suppose $b_i = \beta_{(N)}$ so that i is currently excluded and earns 0. Suppose i increases her bid to β'_i to be included in the market. This changes the lottery to one where the price is $\beta_{(N-1)} > b_i$ with probability q and $\sigma_{(N)}$ with probability $1 - q$, thus violating the NLC condition for buyer i.

Now consider profile $(\beta, \sigma)_j$ and let $k = \min\{N, M\} = N$ (the argument is again analogous when $\min\{N, M\} = M$). Then $\pi_{(\beta,\sigma)_j}$ is a lottery with $p = \beta_{(N)}$ with probability q and $p = \sigma_{(N)}$ with probability $1 - q$ but the buyer with bid $\beta_{(N)}$ and the seller with ask $\sigma_{(N)}$ stay out while all other agents trade. If $s_j < \sigma_{(N)}$, then j cannot affect the price without herself becoming a marginal seller who would earn 0. But as $s_j < \sigma_{(N)} \leq \beta_{(N)}$, the current lottery $\pi\left((\beta, (\sigma_{-j}, s_j)\right)$ yields payoffs $\beta_{(N)} - s_j > 0$ with probability q and $\sigma_{(N)} - s_j > 0$ with probability $1 - q$. So suppose $s_j \geq \sigma_{(N)}$ so that seller j is currently excluded. If j bids some $\sigma_i < s_j$ to be included, then in the best case scenario, she faces a lottery that assigns probability $1 - q$ on $\max\{\sigma_j, \sigma_{(N)}\}$ that either makes j indifferent with the deviation or violates her NLC condition. Thus we have shown that no profile with $k = \min\{N, M\}$ is manipulable.

Case 2 $k < \min\{N, M\}$ (Criterion 2): Consider any profile $(\beta, \sigma)_i$ with $k < \min\{N, M\}$. First consider the case under Criterion 2(B) where either $\beta_{(k)} < \frac{\beta_{(k+1)}+\sigma_{(k+1)}}{2}$ or $\frac{\beta_{(k+1)}+\sigma_{(k+1)}}{2} < \sigma_{(k)}$ as this case is similar to the first case studied. This generates the lottery $\pi_{(\beta,\sigma)_i}$ such that all buyers with bids $\geq \beta_{(k-1)}$ and all sellers with asks $\leq \sigma_{(k-1)}$ trade at price $\beta_{(k)}$ with probability q and $\sigma_{(k)}$ with probability $1 - q$, while all other players stay out. As before, if $b_i \geq \beta_{(k-1)}$ then there is no profitable deviation as any such change can either not affect the price lottery or can throw the buyer out of the market, while the current price lottery satisfies NLC. If $b_i < \beta_{k-1}$, then i can only consider entry by bidding $\beta_i > b_i$, but in each such case, it is straightforward to see that the price lottery would violate NLC for i. Similarly, consider any profile $(\beta, \sigma)_j$ and with $k < \min\{N, M\}$ with either $\beta_{(k)} < \frac{\beta_{(k+1)}+\sigma_{(k+1)}}{2}$ or $\frac{\beta_{(k+1)}+\sigma_{(k+1)}}{2} < \sigma_{(k)}$ as before. If $s_j \geq \sigma_{(k-1)}$ then j is currently out of the market and the only way to enter is by bidding $\sigma_j < \sigma_{(k-1)} \leq s_j$ that yields a strictly negative payoff with probability $1 - q$ and hence violates NLC. So suppose $s_j < \sigma_{(k-1)}$. Then any deviation either has no impact on prices or throws j out of the market.

Next consider the case under Criterion 2(A) where $\beta_{(k)} \geq \frac{\beta_{(k+1)}+\sigma_{(k+1)}}{2} \geq \sigma_{(k)}$, where all buyers with bids $\geq \beta_{(k)}$ and all sellers with asks $\leq \sigma_{(k)}$ trade at price $p = \frac{\beta_{(k+1)}+\sigma_{(k+1)}}{2}$ and all other agents stay out. Let the profile under consideration be $(\beta, \sigma)_i$. Suppose $b_i > \beta_{(k)}$. Then i cannot find a deviation that can affect the price without throwing her out of the market. So suppose $b_i = \beta_k$. By bidding below, she can only throw herself out of the market or keep the price unchanged, while bidding above has no impact. So let $b_i < \beta_k$ so that buyer i is currently out of the market. The only impactful deviations are $\beta_i > b_i$. Suppose first that $b_i = \beta_{k+1}$ and assume $\beta_i > \beta_{(k)}$ (the argument is analogous for $b_i \leq \beta_{(k+2)}$). Note that before this manipulation attempt, it must be that $b_i = \beta_{k+1} < \frac{\beta_{(k+1)}+\sigma_{(k+1)}}{2}$ since as non-trading agents, it must be that $\beta_{k+1} < \sigma_{(k+1)}$, for otherwise they would be trading as marginal agents. Then, after i jumps over $\beta_{(k)}$, the original marginal buyer, pushing this original marginal buyer out of the market, the new price becomes $\frac{\beta_{(k)}+\sigma_{(k+1)}}{2} > \frac{\beta_{(k+1)}+\sigma_{(k+1)}}{2} > \beta_{(k+1)} = b_i$, violating NLC. So assume that i's deviation is such that $\sigma_{(k+1)} \leq \beta_i < \beta_{(k)}$, thereby including himself in the market. If this deviation implies that $\beta_i \geq \frac{\beta_{(k+2)}+\sigma_{(k+2)}}{2} \geq \sigma_{(k+1)}$, then the new price will be $\frac{\beta_{(k+2)}+\sigma_{(k+2)}}{2}$. And this will be beneficial if and only if $b_i = \beta_{(k+1)} > \frac{\beta_{(k+2)}+\sigma_{(k+2)}}{2}$. But this implies this deviation with this resulting new price is feasible and incentive compatible if and only if $\beta_{(k+1)} > \frac{\beta_{(k+2)}+\sigma_{(k+2)}}{2} \geq \sigma_{(k+1)}$, a contradiction with the fact that $\beta_{(k+1)} < \sigma_{(k+1)}$. On the other hand, if this deviation implies either $\beta_i < \frac{\beta_{(k+2)}+\sigma_{(k+2)}}{2}$ or $\frac{\beta_{(k+2)}+\sigma_{(k+2)}}{2} < \sigma_{(k+1)}$, then this deviation creates a lottery price where $\beta_{(k+1)}$ or $\sigma_{(k+1)}$ is the price with probability q and $1-q$ but this buyer is left out of trade and earns 0. So there is again no profitable deviation. Finally, consider the profile $(\beta, \sigma)_i$. If $s_j < \sigma_{(k)}$, then j cannot affect the price without becoming the marginal seller and earning zero. If $s_j = \sigma_{(k)}$ cannot improve by asking more or asking less as by asking less, the only way to affect price is to become a non-trading seller. The arguments for $s_j \geq \sigma_{k+1} > \sigma_{(k)}$, so that she is currently the non-trading seller, is analogous to the non-trading buyer's case. This completes the proof.

5 Concluding discussion

Numerical examples: We begin with an example to demonstrate what the mechanism implements. Let $q = 1/2$ and suppose there are 4 buyers with valuations $b_1 = 10, b_2 = 8, b_3 = 3, b_4 = 1$ and 5 sellers with costs $c_1 = 0, c_2 = 1, c_3 = 2, c_4 = 8, c_5 = 10$. Here $k = 3 < \min\{4, 5\}$ and $(b_4 + c_4)/2 = 4.5 > b_3, c_3$. Hence it is under Criterion 2(B). So the price lottery is $p = 3$ with probability 1/2 and $p = 2$ with probability 1/2 at which buyers 1 and 2 and sellers 1 and 2 trade while all other traders stay out. The market is minimally inefficient. If at the above profile c_4 drops to 3 while all other values remain the same, then while k still remains 3, we now have $b_3 = 3 > (b_4 + c_4)/2 = 2 \geq c_3 = 2$. Hence we are under Criterion 2(A). The market price is $p = 2$ at which buyers 1,2 and 3 and sellers 1,2 and 3 trade while all other players stay out. The market is efficient. Finally, if the valuations for the buyers are $b_1 = 10, b_2 = 9.5, b_3 = 8.5, b_4 = 8.25$ while the sellers' costs remain as above, then $k = 4 = \min\{4, 5\}$ and we are under Criterion 1. In that case, the price lottery is $p = 8.25$ with probability 1/2 and $p = 8$ with probability 1/2, at which buyers 1,2 and 3 and sellers 1,2 and 3 trade and the market is minimally inefficient. It is straightforward to verify that no trader has a profitable deviation. The driving idea of the mechanism is that if one picks β_k as the price, then sellers can bid below cost without risk of loss, but buyers cannot bid above their values. Similarly, if one picks σ_k as the price, then buyers can manipulate, but sellers cannot with risk of loss.

$\mathcal{M}_q$ *without NLC*: M_q is indeed not strategyproof in a world where traders are not NLC. To see this, consider the case when it yields a price lottery of $\beta_{(k)} = 0.7$ with probability q and $\sigma_{(k)} = 0.5$ with probability $1-q$. Suppose $\beta_{(k-1)} = 0.70001$. Buyer (k) is currently not trading. Suppose this buyer bids 0.70002. Then the buyer enters the market and faces the lottery that yields 0.70001 with probability q and 0.5 with probability $1-q$. His *expected* monetary payoff is $q(0.70001 - 0.70002) + (1-q)(0.7-0.5) = (1-q)0.2 - q0.00001$. To block this deviation, we need $q > 0.99995$. But the profile may be such that $\sigma_{(k-1)} = 0.49999$ and if seller (k) who is currently not selling bids 0.49998, then he enters the market and his expected monetary payoff from the new price would be $q(0.7-0.49999)+(1-q)(0.49999-0.49998) = 0$. To block this deviation, we would require $q < 0.00004$. Of course we can now find $\beta_{(k+1)}$ and $\sigma_{(k+1)}$ such that $\frac{\beta_{(k+1)}+\sigma_{(k+1)}}{2} \notin [0.5, 0.7]$. For example, take $\beta_{(k+1)} = 0.65$ and $\sigma_{k+1} = 0.9$.

In general it is far from clear whether No-Loss Constraints faced by agents work in favour or against truthtelling. We found that in the context of double auctions it works in favour of truthtelling. In a world with No-Loss-Constrained traders, we have proposed a variant of MDA that, like MDA, satisfies strategyproofness and is as efficient as MDA, but unlike MDA, is also budget balanced with a uniform price. As a matter of fact, the crucial role of No-Loss Constraints in our mechanism makes a uniform price possible through the use of a price lottery, which in turn trivially ensures balanced budget. Without No-Loss Constraints, our uniform-price mechanism is not strategyproof while McAfee's mechanism is.

Our analysis has been in the context of single-unit demands and supplies. In a multi-unit setup, if each unit has the same value or same cost to the respective buyer or seller, then modifications of the mechanism with respect to number of units traded in equilibrium will keep the basic flavour of our main result. When these valuations are different or when the good is not homogenous, matters will get very complex and clearly beyond the present exercise (see Loertscher and Mezzetti (2016) for more on this issue). Finally, note that although we found that in the context of double auctions, No-Loss Constraints work in favour of truthtelling and help a price lottery achieve balanced budget via uniform price, it does not have the power of going the full distance and making a mechanism fully efficient.

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